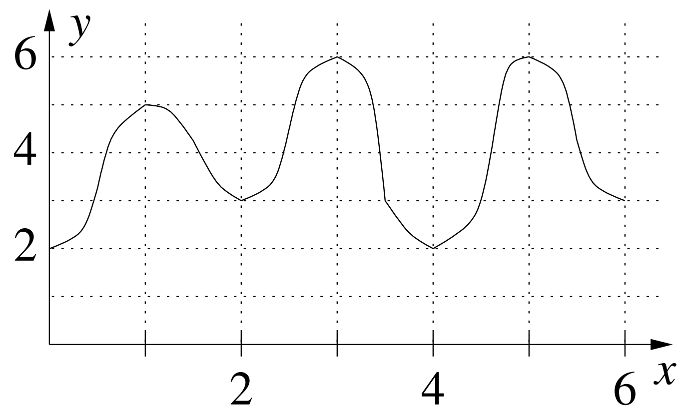
Answers for class prep quiz on section 4.1, Stewart's Calculus (8th ed.)



1. **Answer:** (d). We have $f(0) = 2$ and $f(4) = 2$, and no other value of $f(x)$ is ≤ 2. Note that we can have multiple absolute minima, and that endpoints can be absolute minima. (Same goes for maxima.)

2. **Answer:** (c). Note that the graph of $f(x)$ "peaks" at $x = 1, 3, 5$, and even though $f(1)$ is strictly less than $f(3) = f(5)$, f still has a local maximum at $x = 1$. Note also that the question does not ask about the possibility of $x = 0$ and $x = 6$ being local maxima; in this class, endpoints of the domain will always be ignored in discussions of local minima/maxima.

3. **Answer:** (d). For example, for $f(x) = (x-1)^3$, $f'(1) = 0$ (compute it yourself), but f has neither a local min nor a local max at $x = 1$. As for the other statements, (a) follows from the Extreme Value Theorem (p. 278), (c) follows from Fermat's Theorem (p. 279) and the fact that f is differentiable, and (b) follows from Fermat's Theorem, the fact that f is differentiable, and the fact that if f is a function whose domain is a closed interval $[a, b]$, then any absolute max of f that is not at an endpoint of $[a, b]$ must also be a local max.

4. **Answer:** (c). Consider $g(x) = x^3 - x^2 - 5x - 10$ on the domain $[-2, 4]$. Then
$$g'(x) = 3x^2 - 2x - 5 = (3x-5)(x+1),$$
so the only critical numbers of g are $x = \dfrac{5}{3}$ and $x = -1$. Then because the domain of g is a closed and bounded interval, it remains only to check

the values of g at $x = -2$, $x = -1$, $x = \frac{5}{3}$, and $x = 4$. We get:

$$\begin{aligned} g(-2) &= -12, & g(-1) &= -7, \\ g\left(\frac{5}{3}\right) &= -\frac{445}{27} \approx -16.48, & g(4) &= 18. \end{aligned}$$

Comparing values, we see that g attains an absolute minimum at $x = \frac{5}{3}$ and an absolute maximum at $x = 4$.